Characterization of BandPass Delta Sigma Modulators in Wireless Transceivers Using Parameter Identification

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Abstract **– In this paper, a method to study and characterize a single-loop, cascaded and 1-bit Band-Pass Delta Sigma modulator for digital transmitter is presented. This technique is based on a combination of digital filter simulation and nonlinear optimization of signal-to-quantization noise. The optimal coefficients of modulator structure are achieved by minimization of a quadratic criterion based on prediction error between desired digital filter and noise transfer model. To demonstrate the effectiveness of this approach, simulated results for a 6th order cascaded structure for WCDMA Band-1 standard are presented. Spurious and ACLR improvement could be achieved for this standard with the proposed characterization technique.**

Keywords **– Modulators; optimization; parameters identification; non-linear programming technique; WCDMA.**

1. Introduction

Band-Pass Delta Sigma (BP$\Delta\Sigma$) modulators are more attractive for converting analog radio signals to digital. However, a practical study and analysis of usual modulators topologies finding the optimum $\Delta\Sigma$ parameters to meet requirement for any specific noise shape, does not exist. In literature, the conventional analysis methods are based on the decrease of the quantization noise around the center frequency, so improving the Signal-to-Noise Ratio (SNR) [1][2]. Another technique consists in modeling the quantizer by a gain block and adding a white noise signal that represents the quantization noise process [3]. Although, the imperfections of this model make it impossible to study the modulators properties like stability, implementation possibility and effects of an overload input level [4].

In this paper, our intention is to find the optimal $\Delta\Sigma$ coefficients that yield maximal performance for an usual RF standard. The proposed method is based on parameter estimation by minimization of quadratic error between an ideal filter and noise transfer function. Ideal filter will be designed according to the noise shape specification obtained from a generic frequency response of duplexers and the RF standard. In this case, the $\Delta\Sigma$ coefficients are calculated to minimize the mean-square error based on time domain data generated by a desired digital filter. Performances studied are the Adjacent Channel Leakage power Ratio (ACLR) and the margin with the considered standard spurious.

This technique has been validated by simulation under ADS and MATLAB/SIMULINK software for the optimization of 6^{th} order BP$\Delta\Sigma$ modulator with a WCDMA Band-1 applications. The interest of this RF standard is motivated by the high spurious requirements at different frequency bandwidth.

Section (2) describes the BP$\Delta\Sigma$ architecture and a mathematical model used in the optimization method. Section (3) discusses the estimation of the modulators coefficients by a Non-Linear Programing technique and the definition of the quadratic criterion to be minimized. Section (4) describes the simulation results for a WCDMA standard (Band-1) in term of ACLR and spurious specifications and Section (5) summaries the results and their implications.

2. Digital transmitter based on BP$\Delta\Sigma$ modulator

Fig. 1 shows an usual wireless transmitters using BP$\Delta\Sigma$ modulators [5][6].

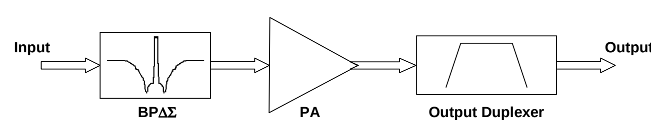


Figure 1. Block diagram of conventional transmitter implemented with BP$\Delta\Sigma$ modulator

In this scheme, an RF modulated signal must be generated prior to transmission. Most commonly, this is done either by generating an analog baseband or intermediate frequency (IF) version of the input signal and then upconverting the signal to RF format. In all transmitters with digital modulation formats, a duplexer is introduced before transmitting the signal via the antenna.

2.1. BP$\Delta\Sigma$ modulator topology

A discrete time BP$\Delta\Sigma$ design requires to choose different elements like discrete resonator cell (z^{-1} or z^{-2}), structure (*Butterworth* or *Tchebychev*) and form (*Cascade-of-Integrators, FeedBack/FeedForward* CIFB/F or *Cascade-of-Resonators, FeedBack/FeedForward* CRFB/F) [7].

In this section, 1-bit quantizer, single-loop and 6$^{\text{th}}$ order CRFB form are discussed (Fig. 2). It consists of sixth cascading resonators operating on the delayed version x_{i_n} of the input sequence u_n. After digitization by the quantizer, a first feedback of the output sequence v_n with the coefficients a_i, is used to provide a maximum *SNR* ratio. A second feedback of the analog propagating signals with the coefficients g_i is inserted. These feedback paths allow a frequency asymmetric repartition of the notches in the noise shape [7]. Input signal is modulated at frequency carrier (F_c) and BP$\Delta\Sigma$ is sampled at four times $F_s = 4\,F_c$.

2.2. State space representation

ADC converter is defined by two transfer functions, the Signal Transfer Function (STF) and the Noise Transfer Function (NTF). In this paper, only the NTF function is considered to extract the BP$\Delta\Sigma$ feedback parameters noted a_i and notches coefficients noted g_i. The *CRFB* structure is described in state space model. This representation based on transition matrix allows to describe easily the modulator behavior. For a 6$^{\text{th}}$ order BP$\Delta\Sigma$, the state space model is defined by the following equations[1]:

$$\left\{ \begin{array}{l} \underline{x}_{n+1} = A\,\underline{x}_n + B\,\underline{e}_n \\ y_n = C^T \cdot \underline{x}_n \end{array} \right. \tag{1}$$

with

$\underline{x}_n^T = [x_1\ x_2\ \cdots\ x_6]$ is the transposed state space vector

$\underline{e}_n = \begin{bmatrix} u_n \\ v_n \end{bmatrix}$ is the sampled input vector

$$A = \begin{bmatrix} 1 & -g_1 & 0 & 0 & 0 & 0 \\ 1 & 1-g_1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & -g_2 & 0 & 0 \\ 0 & 1 & 1 & 1-g_2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & -g_3 \\ 0 & 0 & 0 & 1 & 1 & 1-g_3 \end{bmatrix}$$

1. Note that the proposed state space model can be generalized to an n$^{\text{th}}$ modulator order

$$B = \begin{bmatrix} 1 & -a_1 \\ 1 & -a_1-a_2 \\ 0 & -a_3 \\ 0 & -a_3-a_4 \\ 0 & -a_5 \\ 0 & -a_5-a_6 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

The state space diagram is represented in Fig. 3. Using this representation, the discrete STF and NTF functions can be achieved according to the matrix relation

$$[\,STF \quad NTF\,] = C^T\,(z\,I - A')^{-1}\,B + D \tag{2}$$

where $A' = A + B \cdot [\,0\ \cdots\ 1\,]$ and $D = [0\ \ 1]$

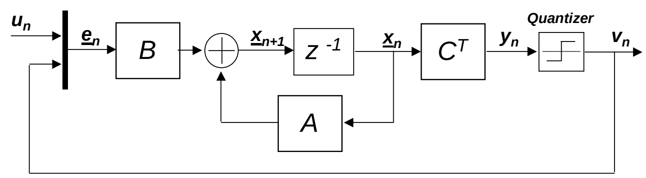


Figure 3. Block diagram of corresponding state space representation

3. Optimal parameters for noise-shaping specification

In order to choose the coefficient values a_i and g_i of the BP$\Delta\Sigma$ modulator, an optimization approach is used: The noise power spectral density obtained from transient behavioral simulations of a desired NTF function is fitted to the previous state space model in the time domain. The optimal fitting is obtained by minimization of the error between the BP$\Delta\Sigma$ model and the desired NTF function. This minimization is based on Non-Linear Programming technique allowing the extraction of an optimal coefficient values.

3.1. Parameter identification algorithm

Parameter estimation is the procedure which allows the determination of the mathematical representation of a real system from experimental data [8]. The block diagram of parameter identification with Output Error technique is shown in Fig. 4. This technique is based on minimization of quadratic error in time domain between required digital filter and NTF function of BP$\Delta\Sigma$ modulator.

For the case of 6$^{\text{th}}$ BP$\Delta\Sigma$ optimization, the previous state space model is considered (Eq. 1) and the following parameter vector is defined:

$$\underline{\theta} = [\,a_1\ \ a_2\ \ \cdots\ \ a_6\ \ g_1\ \ g_2\ \ g_3\,]^T \tag{3}$$

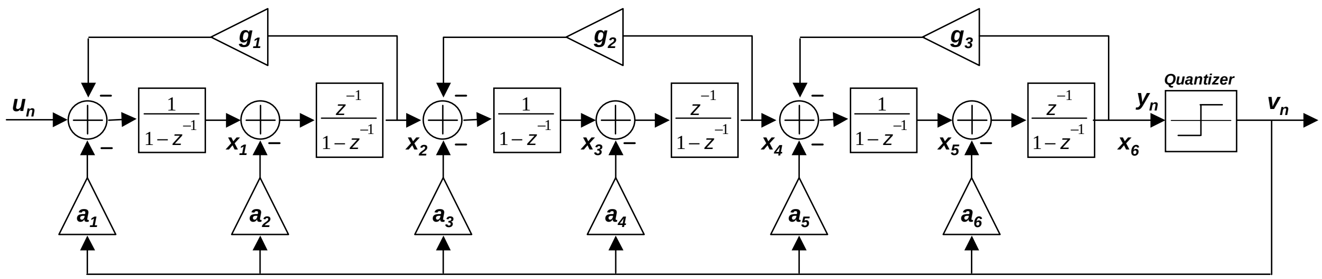


Figure 2. The 6th order BP$\Delta\Sigma$ modulator with CRFB structure

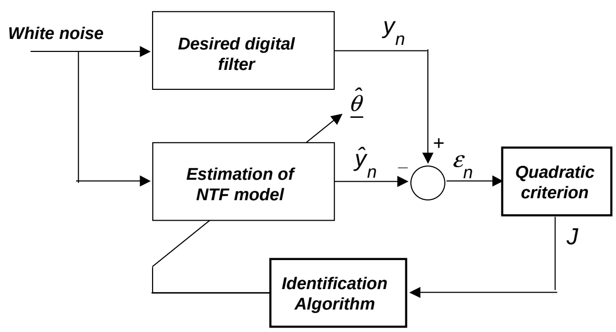


Figure 4. BPDS optimization scheme

Assume that we have measured K values of time-domain input-output $(v(t), y(t)$ with $t = \frac{n}{F_s}$ is the sampled time), the identification problem is then to estimate the values of the parameters $\underline{\theta}$. In practice, the input and output data are obtained by simulation of the desired NTF filter with a white noise uniformly distributed over $[-1, 1]$. Thus, the output prediction error is defined as follow:

$$\varepsilon_n = y_n - \hat{y}_n(\hat{\underline{\theta}}, v) \tag{4}$$

where $\hat{y}_n$ and $\underline{\theta}$ are respectively the estimation of output signal and parameter vector.

As a general rule, parameter estimation with Output Error technique is based on minimization of a quadratic criterion defined as:

$$J = \sum_{n=1}^{K} \varepsilon_n^2 = \sum_{n=1}^{K} (\, y_n - \hat{y}_n)^2 \tag{5}$$

Optimal values of $\underline{\theta}$ are achieved by Non Linear Programming methods. Practically, Marquardt's algorithm [9] is used for off-line estimation:

$$\hat{\underline{\theta}}_{k+1} = \hat{\underline{\theta}}_k - \{[J''_{\theta\theta} + \lambda \cdot I]^{-1}.J'_{\theta}\}_{\hat{\underline{\theta}}=\underline{\theta}_k} \tag{6}$$

with
$J'_{\theta} = -2 \cdot \sum_{n=1}^{K} \varepsilon_n^T \cdot \underline{\sigma}_{n,\underline{\theta}}$: gradient.
$J''_{\theta\theta} \approx 2 \cdot \sum_{n=1}^{K} \underline{\sigma}_{n,\underline{\theta}} \cdot \underline{\sigma}_{n,\underline{\theta}}^T$: hessian.
λ : monitoring parameter.
$\underline{\sigma}_{n,\underline{\theta}} = \frac{\partial \hat{y}}{\partial \underline{\theta}}$: output sensitivity function.

3.2. Desired filter model

When noise-shaping modulators is considered for digital transmitter applications, two performance criterias can be distinguished to specify the desired NTF function:

- the maximum SNR achievable and the possible overload level with the selected topology. These parameters are most important characteristics of the converter because they defined the maximum input signal amplitude for which the structure still operates correctly,
- the requirements of communication standard to be used. Each standard provides some requirements like spurious specifications, frequency bandwidths, transmit (TX) and receive (RX) band. For transceiver design, it is necessary to take into account these criterias.

4. Simulation results

Our objective in this section is to find an optimal coefficients of a 6th order BP$\Delta\Sigma$ modulator for a WCDMA Band-1 norm.

4.1. WCDMA requirements and desired NTF filter design

Among the WCDMA standard bands, Band-1 was selected as it has many spurious requirements defined in different frequency bands, and a gap of 130MHz between Transmit and Receive bands. WCDMA band-1 output power, ACLR and spurious specifications for Tx frequency carrier are detailed on Table 1.

First, a discrete Tchebychev StopBand filter is designed with MATLAB/SIMULINK SOFTWARE according to the standard specifications. Notche positions are defined by poles placement in the unit disk to satisfy the stability condition [7]. However, the noise shaping of the desired *NTF* function is obtained with the zeros placement. For an eventual

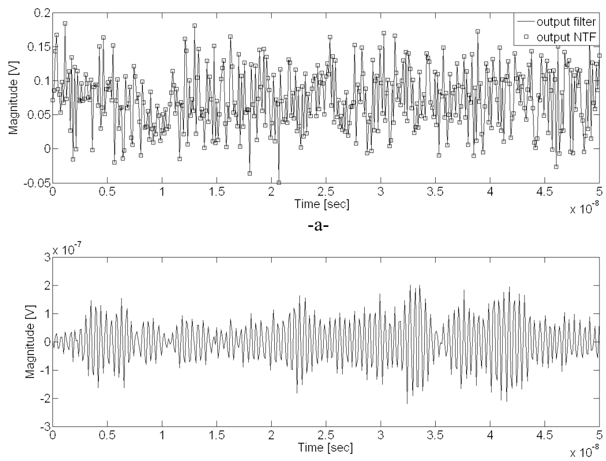
Table 1. WCDMA band-1 requirements

Specifications	Values	
Carrier Freq. (Tx band)	1920-1980MHz	
Output Power	24dBm (+1/-3 dBm)	
ACLR	$F_c\pm$ 5MHz	$F_c\pm$ 10MHz
	33dB	43dB
Spurious	1GHz $\leq$ f $\leq$ 12.5GHz	-30dBm/1MHz
	1.805GHz $\leq$ f $\leq$ 1.88GHz	-71dBm/100kHz
	1.8449GHz $\leq$ f $\leq$ 1.8799GHz	-60dBm/3.84MHz
	1.8845GHz $\leq$ f $\leq$ 1.9196GHz	-41dBm/300kHz
	2.11GHz $\leq$ f $\leq$ 2.17GHz	-60dBm/3.84MkHz

implementation, a commercially available duplexers, implemented for WCDMA band-1, is introduced in the transmitter structure.

4.2. Estimation results

The proposed parameter identification method is used to adjust iteratively the BP$\Delta\Sigma$ coefficients. Appropriated initial values, noted $\underline{\theta}_0$, are required to ensure convergence of the identification procedure. In this case, $\underline{\theta}_0$ can be inserted for the maximum SNR achievable with the topology [4].



-a-

-b-

Figure 5. Comparison between desired and estimated output in time-domain

In time domain, Fig. 5-a shows the comparison between the outputs of the desired filter and the estimated one during optimization. The identification residuals (Fig. 5-b) confirms that the BP$\Delta\Sigma$ behavior is in agreement with the response of the desired digital filter. The achieved coefficients, with Marquardt's algorithm (Eq. 6), are presented in Table 2.

Fig. 6 shows the output spectrum plotted with initial coefficients, estimated coefficients and WCDMA requirements. The result clearly shows that the NTF is optimized to satisfy the standard specifications.

For more evaluation, simulations are done with ADVANCED DESIGN SYSTEM software using an input signal

Table 2. Initial and optimized coefficients

	i	1	2	3	4	5	6
$\underline{\theta}_0$	a_i	0.0428	-0.0437	-0.2468	0.0026	-0.5556	0.5556
	g_i	1.9952	2	2.0047	–	–	–
$\underline{\hat{\theta}}$	a_i	-0.0307	-0.0861	-0.3579	-0.0028	-0.4612	0.6021
	g_i	1.7941	2.2532	1.9524	–	–	–

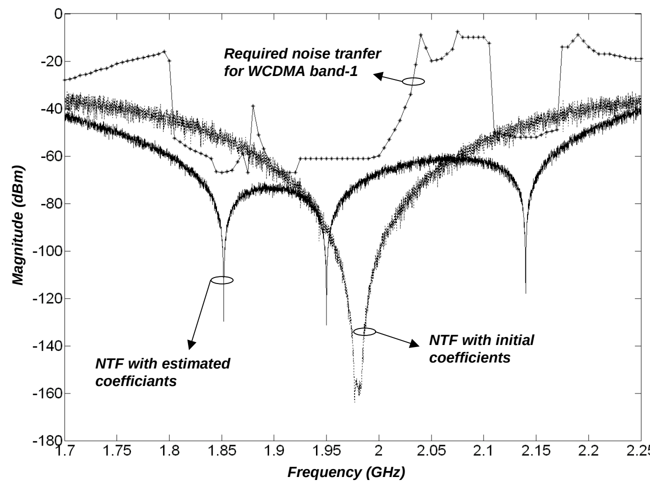


Figure 6. Comparison between initial, estimated and required output spectrum

with a frequency carrier of $F_c = 1.98$GHz and a power of -26dBm, sampling frequency of $F_s = 7.92$GHz and a commercial duplexer. Fig. 7 shows the filtered BP$\Delta\Sigma$ output spectrum with different bandwidth resolutions and the corresponding spurious bands for WCDMA band-1 standard.

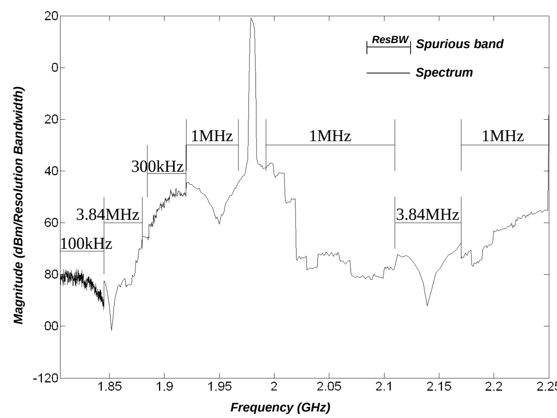


Figure 7. Output spectrum with different resolution bandwidth

As can be seen, the BP$\Delta\Sigma$ spectrum has been correctly shaped and would have 10dB margin with the spurious emission requirements. However we can notice that the simulated structure would still generate significant amount of noise in the receive band. Output power in the channel equal 24dBm.

Table 3. ACLR values

Frequency	1.97GHz	1.975GHz	1.985GHz	1.99GHz
$\mathrm{ACLR}_{\mathrm{required}}$	43	33	33	43
$\mathrm{ACLR}_{\mathrm{achieved}}$	61.34	55.43	53.3	55.43

ACLR results, in dB, given in Table 3, demonstrate that this criteria is respected with a minimum margin of 12.4dB in all TX bands.

5. Conclusion

A new method for BP$\Delta\Sigma$ modulators optimization has been developed. It has allowed us to find the best coefficients set for CRFB topology with respect to RF standard requirements. This procedure is based on output error approach allowing the parameters estimation according to quadratic criterion. The BP$\Delta\Sigma$ feedback and notch parameters are iteratively corrected to satisfy the spurious and ACLR requirements of the WCDMA band-1 standard. The method can be implemented for analog/digital, Lowpass/Bandpass and generalized to n^{th} modulators order.

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